Empirical Analysis of Algorithms for Micro-Planning of Retail Space in Motion Picture Industry

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Technology

by

Amit Kumar Rana

To the

DEPARTMENT OF INDUSTRIAL AND MANAGEMENT ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

June, 2004

CERTIFICATE

It is certified that the work contained in this thesis entitled "*Empirical Analysis of Algorithms for Micro-Planning of Retail Space in Motion Picture Industry*" has been carried out by **Mr. Amit Kumar Rana (Roll No. Y211403)** under my supervision and the work has not been submitted elsewhere for a degree.

June 23, 2004

Dr. Sanjeev Swami
Assistant Professor
Industrial and Management Engineering
Indian Institute of Technology
Kanpur-208016

ACKNOWLEDGEMENT

I wish to express my deep sense of gratitude and indebtedness towards Dr. Sanjeev Swami for his inspiring guidance, invaluable suggestions and constructive criticism. He was always a constant source of encouragement throughout my thesis work, and was more than a thesis supervisor.

I would also like to extend my sincere thanks to Anirudh, Manish, Babu, Vijay, Aashish, Akhilesh, Tripti, Priyanka, Avinash, Vaibhav and others for making my stay at IITK, a smooth and memorable one.

I extend my thanks to IME faculty members and staff for showing a special bondage towards me as an IME family member.

Last but not least, I am grateful to my respected parents and the almighty for providing me courage and spirit to accomplish this exigent task.

- Amit Kumar Rana.

CONTENTS

List of Figures and Tables

ABSTRACT

Retailing is becoming an increasingly important area of management attention and academic research. The development of decision support models to help retailers improve their decision-making is gaining importance. Every week motion picture retailers, i.e., exhibitors, have to make an important decision regarding the replacement of the movies playing at the screens in their theaters. The dynamic and uncertain decision environment, complicated contract terms, a number of new products and their perishing demand give rise to the complexity of the problem. In most instances, this complexity is further increased by availability and choice of multiple display facilities.

This thesis is related to the micro-management of movie scheduling. In micro-scheduling of movies, we consider the problem of choosing at what times and on which screens of a multiplex theater to play movies during a day. Under reasonable assumptions, we show integer linear programming is sufficient for formulating this system. We present actual schedules followed by a theater manager in The Netherlands from January 10, 2002 to January 16, 2002 on the basis of given demand data points. These are compared with optimal schedules that are given by the mathematical models of varying complexity based on different evaluation parameters. We then investigate the improvement shown by the proposed models over actual schedule and robustness of results of optimal schedule through sensitivity analysis.

CHAPTER 1

INTRODUCTION

Retailing, one of the important functions of marketing, is becoming an increasingly important area of management attention and academic research, especially in the developed countries. One of the major areas of research in retailing is the development of decision support models to help retailers improve their decision-making in the dynamic and complex environment. Most previous marketing research in this regard has been concerned with consumer durable goods, not with services or intangibles. This thesis seeks to advance our understanding of how quantitative model building can improve marketing decision making in complex dynamic environments by focusing on a perishable entertainment product, namely, movies. However, our methodology and results can readily be generalized to other entertainment products (e.g., performing arts, books, video games), travel services, fashion goods, and educational programs.

A movie is an interesting product for several reasons. Most movies are separate entities and can be considered innovations because of the new features (e.g., actors, storyline, music, etc.) usually included in them. These new movies are released every week throughout the year. The product life cycle of movies is relatively short and is measured in weeks. The product is seasonal in nature and the prominent seasons are during holidays. Finally, one of the most interesting attributes of movies from a research standpoint is their perishability, which is time based deterioration of demand /appeal of a movie. Thus a theater owner, with an objective of effective screen management, faces a complex scenario. The complexity comes from various sources. First, the relatively large

number of movies available ("Too many pix, too few screens," Variety 1995) combined with a short and decaying audience appeal over time poses a complex management challenge. The decision is further complicated because it is made for a number of screens in a multiple theater (i.e., a multiplex). Second, each week's release of new movies brings continual pressure from the distributors to generate screens and playing for them. Third, exhibitors often possess a number of facilities (i.e, theater) in the same geographical area. This presents another booking challenge of managing the interdependency among several facilities. Fourth, the nature of the distributor-exhibitor contract in the motion picture, in the U.S as well as in Europe, is unique. For example, in the USA, in signing a contract to play a movie in its theaters, the exhibitor becomes obligated to play the film for a certain period of time even when audience demand is weak. The financial arrangements between distributors and exhibitors are also unique to the motion picture industry. Box office receipts are split between the distributors and exhibitors such that the split favors the distributor in the first few weeks of movie playing, but shifts to the exhibitor's favor later on. Distributors thus have a strong incentive to promote the movie intensively in their initial play period. On the other hand, the longer the exhibitor plays the movie, the larger is its share of the box office receipts becomes. At the same time, theatre attendance for a movie typically declines the longer it plays. Generally, all concession revenues are retained by exhibitor.

The complexity of the screen management problem just described indicates that there is a strong need for a marketing management support system that can help theater programming managers in their task of optimally choosing movies for their limited screen capacity. These marketing management support systems (MMSS) have a high

potential for helping managers, but an unpredictable chance to succeed. While many of its managerial problems tend to be fairly structured, the decision environment is quite dynamic, contractual arrangements between parties are complex, management turnover appears to be high, and perhaps most importantly the cognitive style of the decision makers is often non–analytical or heuristic in nature (Wierenga, van Bruggen, and Staelin 1999). These characteristics represent challenges in developing implementable MMSS, though other areas of the arts and entertainment industry (e.g., Weinberg 1986) provide some optimism for the movie industry.

In response to the dynamics and challenges posed by the above characteristics of movies, a stream of research, particularly addressing the marketing of movies, has contributed to the marketing literature. At the consumer behavior level, some of the research has questioned the relevance of the traditional information-seeking framework for studying the consumption of movies (e.g., Hirschman and Holbrook 1982; Holbrook and Hirschman 1982). Another stream of research has focused on forecasting the enjoyment of movies at the individual level (Eliashberg and Sawhney 1994) as well as forecasting the commercial success of movies at the aggregate level (Smith and Smith 1986; Austin and Gordon 1987; Dodds and Holbrook 1988; Sawhney and Eliashberg 1996; Eliashberg and Shugan 1997). Additionally, some research has begun to emerge addressing diffusion (Mahajan, Muller, and Kerin 1984; Jones and Ritz 1991), seasonality (Radas and Shugan 1995), release timing (Krider and Weinberg 1998), clustering (Jedidi, Krider, and Weinberg 1998), sequential products (Lehmann and Weinberg 1998; Prasad, Mahajan, and Bronnenberg 1998), contract design (Swami, Lee, and Weinberg 1998), scheduling (Swami, Eliashberg and Weinberg 1999; Eliashberg,

Swami, Weinberg and Wierenga 2001) and the impact of advertising (Zufryden 1996). This thesis advances the above stream of research in marketing of movies by considering the problem related to retailing of movies.

Typically, perishability is thought of in terms of physical deterioration of a product such as a grocery item with an expiry date. In some sense, this view comes from the supply side of the product. In contrast, in this thesis we adopt a "demand side view" in the context of perishability. Thus, the physical product in the problems considered remains the same, but its demand perishes over time. In recent years considerable work has been done on the treatment of perishability in inventory control (e.g., Abad 1996, Jain and Silver 1994). In inventory control, perishability refers to the physical deterioration of units of a product. For the movies, we analogously define perishability as the decrease in the value/appeal of a movie with the passage of time. As mentioned earlier, previous research in shelf space management and dynamic decision-making has mainly focused on non-perishable goods. In the context of movies, the complexity introduced by perishability in dynamic shelf space management problem can be summarized as: which motion pictures to choose to show each week/day and for how long to play them. In case of exhibitors with multiple screens, issues such as allocation of different movies to different capacity screens, switching of the movies between screens and multiple screening of same movie add to the complexity of the general problem. This decision-making problem is further complicated by additional factors specific to the movies context. We discuss such factors in detail in the later chapters.

The exhibitors, who are the retailers in movie market, also face the problem of limited "shelf–space" which is the available screens for scheduling various movies. The

exhibitor problem can be addressed at two different levels. At level 1, the exhibitor has to select the movies, which are to be run on multiple screens in the coming weeks. Basandani (2002) assumed an exponentially declining demand pattern of each movie for this multiple–screen problem. The complexity in such problem is because of multiple unequal capacity screens and the decay of movies, which causes the exponentially declining demand pattern. With an assumption of equal screen capacities, the problem is similar to the convention parallel machine scheduling problems. Some work has been done in this regard particularly focusing on the movie exhibitors (Swami, Eliashberg, and Weinberg 1999). However, this assumption of equal screen capacities poses serious limitation in impletation of such models. Considering different capacities for screens introduces nonlinearity into the resulting model and makes it less tractable (Saxena, 2000). Further, the application of Genetic Algorithms methodology (Holland 1975, Michalewicz 1992, Grefenstette 1986) based heuristic (Moholkar 2001) showed improvement over other heuristics. Basandani (2002) added additional constraints like commitment and obligation period to Moholkar's (2001) approach.

At level 2 of the exhibitor's problem, labeled micro-scheduling, and addressed specifically in this thesis, we propose several models to assist the theater manager to decide the slots and screens in which movie should run during a day. The theater opens up at 10:30 hrs in the morning and the closing time is 00:10 hrs. For modeling purposes, the period between 10:30 to 00:10 hrs is divided into eighty-three slots of ten minutes each. The whole day is divided into two sessions, namely, morning and evening session. The morning session refers to the period from the first slot to the forty-fifth slot and the evening session refers to the period from the forty-sixth to the end of the day. Further, the

period from thirty-seventh to the fortieth slot is designated as the break period, during which no new movie should start.

The objective of the theater owner is to generate a movie schedule to earn maximum revenue while satisfying the following constraints:

a) At each time slot, at most one movie is running on each screen.

b) At each time slot, each movie should run on at most one screen.

c) No movie should open before 10:30 hrs and all the movies should close before or at 00:10 hrs.

d) No movie should start during the break period.

e) There should be at least twenty minutes break (called cleaning time) between the ending and the starting time of two consecutive movies on the same screen.

The above set of constraints is collectively called the basic constraint set. In addition to the basic constraints, there are several additional managerial constraints explained below.

a) **Every 20-Minute constraint:** At least one movie should start every twenty minutes.

b) **High-Low constraint:** During the evening session, for each time slot, no more than one movie should start on the high-capacity and low-capacity screens, Screens 1 to 8 are labeled as low-capacity Screens, while screens 9 to 13 are labeled as high-capacity screens.

c) **Hopping constraint:** A movie should stay on the same screen once allotted, for the morning and evening sessions. If a movie switches from allotted screen within the morning and evening sessions then it is called a hop. A penalty is associated with Screen hopping of the movie and is called the hop-cost.

The aforesaid model is tested on the data for the period from January 10, 2002 to January 16, 2002 obtained from De Munt Theatres, The Netherlands. The data includes information about the run-length of the movies, number of screens, capacity of the individual screens and the movie schedule (demand data points). The slot-by-slot demand matrix is generated by extrapolating and interpolating the demand points (obtained from the actual schedule).

This research addresses the following five versions of the micro-scheduling problem:

- **Basic:** Addresses only the basic constraints.
- **Basic-High-Low:** Addresses the basic constraint set and the managerial constraint that during the evening session, no more than one movie should start on the screen 1 to 8 and 9 to 13.
- **Basic-20Minutes:** In addition to the basic constraint set, it includes the managerial constraint that at least one movie should start every twenty minutes.
- **Basic-Hop:** Additional constraint to basic constraints ensuring that once a movie is allocated to a screen during the morning or evening sessions, it should stay on the same screen. If a movie switches from allotted screen during the morning and evening sessions then it is called hop. A penalty is associated with this hop, which is called the hop-cost.
- **Basic-Hop-High-Low:** Addresses the combination of Basic, Hop and the High-Low constraints.

The rest of the thesis is organized as follows. Chapter 2 reviews the relevant literature. Chapter 3 explains how the scheduling decisions are taken in motion picture industries and briefly explains the problems faced by the exhibitors. This chapter also

discusses the SilverScreener model proposed by Swami, Eliashberg, and Weinberg (1999), which addresses the macro-scheduling problem of exhibitors with multiple screens with assumptions of deterministic demand pattern of movies and equal capacities of all screens.

Chapter 4 explains the various conceptual models of varying complexity, proposed in this research. In this chapter, we discuss the various issues, which are being faced by the exhibitors of multiplex theater in scheduling the movies on different screens of the theater during a given day and explain the demand extra/intra-polation approach.

In Chapter 5, we explain the procedure for empirical analysis and sensitivity analysis. In empirical analysis, comparison of the optimal schedules, generated by the conceptual models by changing complexity with actual schedules is done based on different parameters. Then, we investigate the improvement shown by our models over actual schedule and robustness of results of optimal schedule through sensitivity analysis.

Chapter 6 explains the results and discussion of empirical and sensitivity analysis. In this chapter, we conclude by discussing the limitations of the current research and directions for the future research.

CHAPTER 2

LITERATURE REVIEW

Previous researchers have recognized the interface between marketing and operations as an important research domain in the quantitative modeling research stream. In the operations management literature, Acquilano and Chase (1991, p. 17) mention that marketing specialists need an understanding of what the factory can do relative to meeting customer due dates, product customization, and new product innovation. In service industries, marketing and production often take place simultaneously, so a natural mutuality of interest should arise between marketing and OM [operations management]. Karmarkar (1996) stresses the need to do more integrative research between marketing and operations management. This thesis addresses such needs and uses both marketing and operations management tools in solving management problems.

2.1 Shelf Space Management Problems

Shelf (display) space management is a critical issue to the retailers. First, consumers choose from the products that are displayed on the shelf. In this sense, shelf space management determines the ultimate profitability of the retailer. Second, most retailers have limited shelf space to display their products. Therefore, the choice of which products to display becomes an important retailing decision. The decision is particularly complex in those industries in which many new products are continually introduced in market. Finally, anticipating and adapting to dynamic changes in consumer's tastes and demand are key concerns to most retailers. Moreover, such decisions have to be made for several product categories. Though previous researchers (Corstjens and Doyle 1983;

Bultez and Naert 1988; Borin, Farris, and Freeland 1994) have addressed some of these issues and they have generally been in the context of packaged goods in a supermarket chain setting. However, other industry settings, such as the motion picture (or movie) industry considered in this thesis, pose different challenges and intriguing problems.

2.2 Perishability

One of the interesting and important attributes of the motion pictures is their perishability. In recent years considerable work has been done on the treatment of perishability in inventory control (e.g., Abad 1996, Jain and Silver 1994). In inventory control, perishability refers to the physical deterioration of units of a product. For the movies, we analogously define perishability as the decrease in the value/appeal of a movie with the passage of time. As mentioned earlier, previous research in shelf space management and dynamic decision making has mainly focused on non-perishable goods. In the context of movies, the complexity introduced by perishability in dynamic shelf space management problem can be summarized as: *which motion pictures to choose to show each week and for how long to play them*. This decision-making problem is further complicated by some other factors specific to the movie context. We discuss such factors in detail in the later chapters.

2.3 Integer Programming Approach

A linear programming problem in which some or all the variables must take non-negative integer values is referred to as integer linear programming problem. When all the variables are constrained to be integer, it is called a pure integer-programming problem, and in case only some of the variables are restricted to have integer values, the problem is said to be a mixed integer-programming problem. In some situations each

variable can take on the values of either zero or one; such problems are referred to as zero-one programming problems.

Integer programming is a valuable tool in operations research having a good potential for applications. Such problems occur quite frequently in business and industry. All the assignment and transportation problems are integer-programming problems. In these types of problems the decision variables are either zero or one.

In all such situations, the decision variable X_j,

$$X_j = \begin{cases} 1 & \text{if } j^{th} \text{ activity is performed,} \\ 0 & \text{if } j^{th} \text{ activity is not performed} \end{cases}$$

In addition, all allocation problems involving the allocation of men and machine give rise to integer programming problems, since such commodities can be assigned in integers and not in fractions.

The integer programming problems are solved by either the Cutting Plane or Branch and Bound method. In Cutting Plane method by Gomory (1958), we first solve the integer programming problem as ordinary L.P. problem and then introduce additional constraints one after the other to eliminate certain parts of the solution space until an integral solution is obtained.

In Branch and Bound method, the problem is first solved as a continuous L.P. problem ignoring the integrality condition. If in the optimal solution some variable, say X_j is not an integer, then

$$X_j^* < X_j < X_j^* + 1$$

where X_j^* and $X_j^* + 1$ are consecutive non-negative integers. It follows that any feasible integer value of X_j must satisfy one of the two conditions, namely

$X_j < X_j^*$ or $X_j > X_j^* + 1$.

These two conditions are mutually exclusive and when applied separately to the continuous L.P. problems, form two different sub-problems. Thus the original problem is "Branched" into two sub-problems. Each of these sub-problems is then solved separately as a linear program, using the same objective function of the original problem. If any sub-problem yields an optimal integer solution, it is not further branched. However if it yields a non-integer solution, it is further branched into two sub-problems. This branching process is continued until each problem terminates with either integer valued optimal solution or there is evidence that it can not yield a better one.

2.4 Decision Support Models in Marketing

Over the last three decades, decision support modeling has flourished in marketing. Today one can name many successfully implemented decision support models in marketing, such as PROMOTIONSCAN (Abraham and Lodish 1993), Rangaswamy, Sinha and Zoltners's (1990) model on sales force restructuring, SHARP (Bultez and Naert 1988), ARTS PLAN (Weinberg and Shachmut 1972), BRANDAID (Little 1972), CALLPLAN (Lodish 1971), and so on[1]. These models have addressed various aspects in marketing at different levels of a supply chain. For example, CALLPLAN efficiently allocates sales force to customers and products. ARTS PLAN helps manager plan a series of performing arts presentations. PROMOTIONSCAN helps manager in developing and evaluating short-term retail promotions. SHARP model helps retailers decide on shelf-space allocations. The SilverScreener model (Swami, Eliashberg, and Weinberg 1999),

[1] Many commercial models based on the similar concepts are now also available in the market.

presented in Chapter 4, is similar to PROMOTIONSCAN and SHARP models and its aim is to help retailers (of the motion picture industry) improve their decision-making.

Marketing decision support systems (MDSS) (Little 1979, p. 9) are intended to assist decision-makers in taking advantage of available information. Decision-makers should benefit from the availability of more or better data by incorporating the information derived from these data into their decision processes (Blattberg and Hoch, 1990). For this purpose MDSS contain marketing models that make it possible to perform so-called what-if analysis (Wierenga et al. 1994). Blattberg and Hoch (1990) report that a combination of model and manager often outperforms either of these two alone. Hoch (1994) attributes this to the relative strengths of models, which compensates for the relative weaknesses of managers. In general, some of the ways in which the use of decision support models can aid a marketing executive are the following (Montgomery and Weinberg 1973):

- Helps to better utilize a manager's judgment,
- Requires an explicit listing of input assumptions which leads to more informed discussion,
- Provides a method for quick and convenient evaluation of the consequences of alternative plans,
- Allows the emergence of unexpected solutions which open up new areas of problem-solving,
- Expands the range of questions which can be answered by use of the notion of derived judgment,
- Distills from available data relevant information as in new product forecasting,

- Provides a basis for relating marketing inputs to market results and, hence, serves as basis for marketing planning, and
- Diagnoses based on early data, the adequacy of a market plan and locate areas needing improvement.

Inspite of these benefits, one must recognize the following important points about decision support models. First, models are an aid to the decision-maker, not a replacement. Marketing models often can help the manager make a better decision, but models do not make executive decisions by themselves. Second, models should be proposed as useful tools to the end-user (i.e., manager), not as academic curiosities. Finally, models can be useful to managers in many different ways, and may offer opportunities for efficiency in a broad range of managerial activities.

2.5 Micro Scheduling

Scheduling is the allocation of resources over time to perform a collection of tasks. Scheduling approaches are applicable in different situations, such as classroom scheduling, allocation of available resources for advertisement of new film releases, staff scheduling in any industry, travel services, allocation of available resources in making a new product etc.

As an illustration, the objective of class room scheduling is to choose meeting rooms and times for each class that maximize student and instructor preferences without creating student, room or instructor schedules conflicts. In classroom scheduling, the resources include instructors, classrooms, and group of students or classes. Effective classroom scheduling maximizes the likelihood that students can get desired courses, while considering other goals and constraints such as maximizing facility utilization and

instructor preferences. Good schedules are those that maximize the likelihood that students are able to schedule selected courses. Feasible schedules must not assign instructors or classrooms to more than one meeting simultaneously or to times when they are unavailable. Also, rooms may not have sections assigned that exceed their seating capacity. To solve the large scale classroom scheduling problem, Edwards, Rardin and Parmenter (1996) presented a model. In this model relation between sets of time patterns, classrooms and sections are made explicit with a non-linear 0-1 (Binary) integer programming. This relation is defined by a binary variable $P_{t,s,c}$. It means that if a time slot (t) is allotted to a specific section (s) in a particular classroom (c) then the value of binary variable will be 1 otherwise 0.

In movie industry, a new modeling approach is developed to evaluate the market performance of new film releases as a function of advertising by Zufryden (1996). The proposed model sequentially links planned advertising expenditures for a new film introduction to awareness, intention to see the film, as well as projected ticket sales at the box office. It is illustrated how the model may be used by a movie studio to evaluate alternative film introduction strategies based on proposed allocations of advertising expenditures as well as theater distribution intensities over a film's life cycle. Also, in movie scheduling, some research has begun to emerge addressing contract design (Swami et al. 1998), scheduling (Swami, Eliashberg and Weinberg 1999; Eliashberg, Swami, Weinberg and Wierenga 2001).

CHAPTER 3

SCHEDULING DECISIONS IN MOTION PICTURE EXHIBITORS INDUSTRY

In retailing, shelf space is a fixed resource. Managing this space means making frequent decision about which product to stock and how much shelf space to allocate those products. A lot of research has focused on the development of decision support system to help retailers improve their decision on shelf space allocations. Our research focuses on the products, which have relatively short life cycles, such as movies, so that effective retail management requires regular attention to the issue of which product is to be stocked.

The motion picture industry is emerging as an area of increased interest to marketing scholars and researchers. A stream of research, addressing various aspects related to marketing of movies, has begun to emerge in the marketing literature. At the consumer behavior level, some of the research has questioned the relevance of the traditional information seeking framework for studying the consumption of movies (Hirschman and Holbrook 1982). Another stream has focused on forecasting the enjoyment of movie at the individual level (Smith and Smith 1986, Austin and Gordon 1987, Dodds and Holbrook 1988, Sawhney and Eliashberg 1996, Eliashberg and Shugan 1997). Additionally, some research has begun to emerge addressing diffusion (Mahajan et al. 1984, Jones and Ritz 1991), release timing (Krider and Weinberg 1998), clustering (Jedidi et al. 1998), and the impact of advertising (Zufryden 1996), all in the context of motion pictures.

A theater manager with an objective of screen management faces a complex scenario. The complexity comes from various sources. First, the increased supply of movies by various studios increases the difficulty of deciding which movie to play. This decision is further complicated because it is made for a number of screens. Second an additional supply of movies brings more pressure from the studios to guarantee sufficient play time for their movies. Relationship management in the motion picture industry is considered by many as very crucial. On the other hand, the scarcity of "shelf space" requires special attention in managing the screens effectively and profitably. Third, the nature of the distributor exhibitor contract in the motion picture industry is unique. There may be a contract to play a film in its theater for a certain period, even when the consumer demand is weak later on in that period. This minimum obligation period, which is negotiated between two parties, may vary by movies as well by studio. The financial arrangement between exhibitors and studios are also apparently unique to the motion picture industry. The manner in which the box office grosses are split favors the distributors in the first few weeks of the movie playing and, but shifts to the exhibitor's favor later on. Distributors thus have a strong incentive to promote the movies in their initial play period. On the other hand, the longer the exhibitor plays the movie, the larger becomes his share of the box office receipts. At the same time, theater attendance for a movie typically declines the longer it plays.

In the face of this complexity, the exhibitor problem can be addressed at two different levels. As shown in Figure 3.1, at Level 1, the SilverScreener approach (Eliashberg, Swami, Weinberg and Wierenga 2001) is used for generating the macro schedule. This schedule contains the list of movies, which are to be run in the coming

weeks on different screens of any given theater. The SilverScreener optimization model helps to select and schedule movies for a multiple screens theater over a fixed planning horizon in such a way that the exhibitor's cumulative profit is maximized. It uses an exponentially declining demand curve to estimate the number of visitors attracted by a movie and assumes screens of equal capacity. It uses the rolling horizon approach. This approach allows considering the long run implications of its choices, while still allowing the decisions to be based on the most recent data (about ticket sales, movie availability, and contract terms).

The constraints to the model are that only one movie should be allocated to one screen in one week in each movie week of the planning window, a movie can not be scheduled on more than one screen, the total number of movies scheduled in a given week should be equal to the total number of screens in the theater, and that movie can not be scheduled before its release date and after its due date. This model was initially tested for a single theater multiplex of different capacity screens and the results were satisfactory. Subsequently, the SilverScreener approach was extended to multiple theater scheduling of Pathe theaters in Amsterdam.

In the context of U.S.A, macro scheduling operates as follows. Before a season begins, exhibitors select a set of movies to book and develop a tentative schedule (or "master plan"). Approximately three to four month before the summer season, distributors screen their movies for exhibitors at a major trade show. After the screening distributors send out bid application to most exhibitors. A typical bid invitation contains the release date of the movie, the contract terms (i.e. obligation period and sharing terms) that vary by both movies and distributors, bid return deadline, and so on. Using the

information about the contract term for various movies from the distributors' bid invitation letters, and a suitable revenue prediction scheme, the exhibitor can come up with a tentative preseason schedule for a multiplex using the model. Such a schedule can help the exhibitors decide whether or not bid for a particular movie. The preseason schedule would reject some movie outright, because either their contract terms are too unattractive, or their estimated profit potential is not attractive enough as compared to other movies.

When the season starts, an adaptive scheduling approach, that is repeated weekly application of SilverScreener algorithm, helps the exhibitor to decide how long should a movie play. In practice, the exhibitors decide once a week (normally on Monday), on which movies to play starting later in the week (usually on Friday). This is an adaptive decision making mode of behavior. In such situations, the exhibitor is likely to choose a shorter planning horizon than the one for preseason bidding because he will be more certain of the availability and revenue potentials of the various movies. The exhibitor makes weekly decisions rolling from one time window to another. After a movies plays at the theater (if it is chosen to play), and the corresponding actual revenue data becomes available, the exhibitor can update the revenue prediction based on the new data.

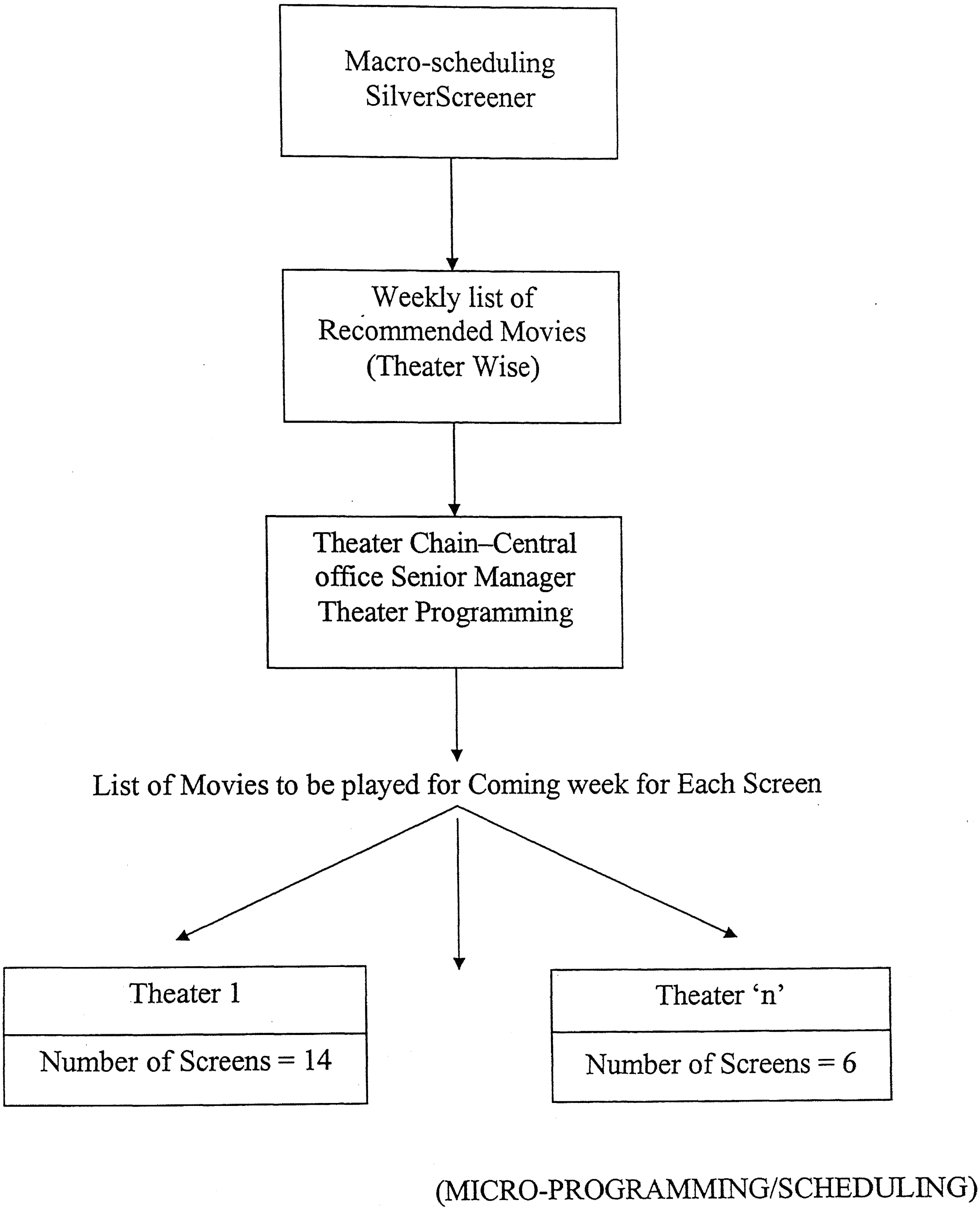

Fig 3.1: Macro and Micro-Planning of Movies in a Multiplex Theater

This thesis is aimed at the Level 2 of the exhibitor's problem labeled as micro scheduling. It investigates several models to assist the theater manager to decide the slots and screens in which movie should run during a day. The optimal macro schedule gives the overall list of movies, which should run in the coming week on different screens of the multiplex theater. The micro scheduling algorithms help the exhibitors to plan the time-slot and screen allocation, and the frequency of the movies selected by macro-scheduling in order to maximize the theater revenue and at the same time, address additional issues such as customer satisfaction.

CHAPTER 4

MICRO–MANAGING MOVIE SCHEDULING PROBLEM

4.1 Introduction

In micro-managing movie scheduling, we consider the problem of choosing the times and the screens in which to play movies. Under many assumptions (which are tractable and discussed later), we show that integer linear programming is sufficient for solving the system.

The schedule generated in macro scheduling gives the list of movies, which should run in the coming week on different screens of the multiplex theatre. The movies recommended to run on the different screens of the theatre in any week have different demand pattern during a day. The demand is also different on weekdays and weekends. The micro scheduling program helps exhibitors to plan which movie he/she should play in which time slot and screens so that he/she can maximize the cumulative revenues.

4.2 Micro Management Problem

There are several factors, which should be taken into account before scheduling the different movies. These are as follows:

1. Movies to be played on various screens are of different lengths (multiple of time slots).
2. Movies can be scheduled multiple number of times in a day.
3. There is more than one screen available.
4. The capacity of each screen is different.

5. At different time slots (start time), movie can generate different revenue.
6. Revenues are assumed to be known for the scheduling problem and can be estimated in advance.
7. The final goal is to maximize the cumulative net contribution (including concession).
8. Movie scheduling must satisfy other administrative constraints.

4.3 Real World Issues in Micro Scheduling

There are many practical issues, which are dealt by manager of any multiplex theater in generating this micro schedule. These are discussed below.

1. *Customer satisfaction*: For the convenience of the customer, the theater would like to schedule the movies in such a fashion that whenever the customer arrives at the theater there should be at least one movie to start with in 20 minutes so that the average waiting time for a customer to watch a movie can be minimized.
2. *Multiple copies of a movie*: The movies, which are in the consideration set of the micro scheduling, may have multiple copies. For the theater, it will not be desirable to schedule these multiple copies running at approximately the same time.
3. *Floor capacity constraint*: The multiplex theater has different floor sections with some floors having more than one screen. Hence, the manager of the theater would not like to start two movies on different screens of the same floor simultaneously to avoid the crowd on that floor exceed the capacity of the floor.
4. *Many big movies at the same time*: The manager of the theater would not like to start many big movies at the same time, as it will need more staff to be hired to

deal with the volume of people arriving for same time showings. If additional staff is not hired and the lines are long, the people may become frustrated, which could affect their satisfaction level at the theater and this unpleasant wait may result in a decreased to return the theater.

4.4 Demand Estimation for Movies in Different Time slots

The movies selected for the screening have different genre such as comedy, family, drama, horror, crime, action, and others. These different types of movies may have different demand pattern over the day. For estimating the demand during different time slots, the following different approaches are adopted.

Approach 1:

When enough data points are available.

Movies like Don't say A word, Ocean's 11, Harry Potter play in every time zone (say >= 3 time points).

In such cases, we do a simple linear interpolation between the given data points.

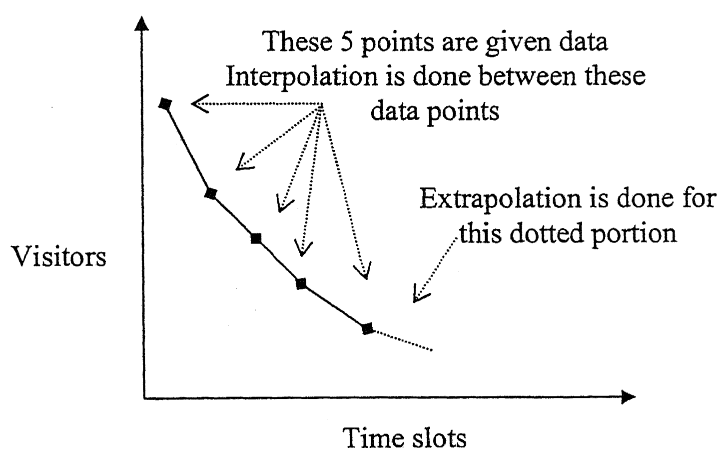

Fig 4.1: Demand Estimation by Linear Interpolation Approach

Approach 2:

When few data points are available.

Ocean's 11 plays in 5 time slots and a movie with similar parameters Bandits plays in 2 time slots at 18:50 and 21:25.

In such cases, we use a "Ratio technique".

We use percentage of Ocean's 11's demand Bandits had at 18:50 and 21:25 (ratio of demand at those two times). Then we calculate the multiplicative factor by average of ratios. After it to estimate demand for Bandits at other points, we multiply factor to demand of Ocean's 11 at the other points at which Ocean's 11 plays, but Bandits does not play.

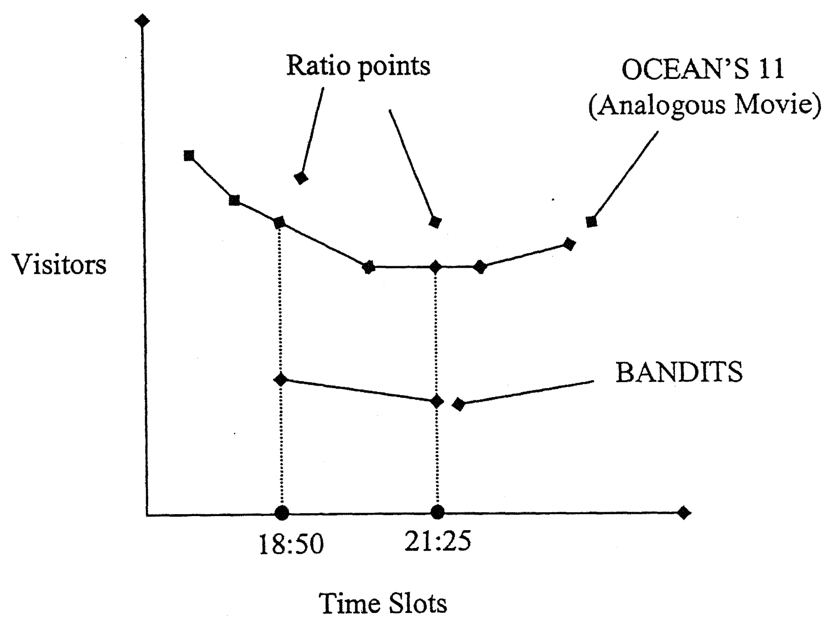

Fig 4.2: Demand Estimation by Ratio Technique

Approach 3:

When there are some restrictions on playing movies after a particular slot.

Few movies like Minoes, Harry potter NL cannot be shown after a particular time slot.

In these cases, we do simple linear interpolation for the given data points and make demand 0 after that particular time slot.

4.5 The Model Formulations

The micro-scheduling problem can be handled in different steps of increasing complexity. Based on the original formulation by Eliashberg, Miller, Swami, Weinberg, and Wiesenga (2003), we begin with the basic model of the problem and then extend the basic model to the other models by adding managerial constraints, which are more complex in nature. Four extensions of the basic model are considered here for modeling and analysis purpose. There are some basic assumptions, which would be followed in each model. These are:

Assumptions:

In order to simplify the exposition, we make the following assumption-

- We assume that the movies, which are to be run, have already been chosen. Given this list of candidate films, we now choose which times and screens to play each movie.
- It takes the same amount of time to clean a screen and get it ready for the next movie regardless of what the next movie is; there is no extra time required to get the screen ready if we switch movies.
- No movie is simultaneously shown on two screens.

Input Data

There are some important inputs, which would be required for the basic model formulation. These are as follows:

- Number of screens in the theatre
- Capacity of each screen
- Number of candidate movies
- Time the theater opens for showing movies
- Time the theater closes for showing movies
- Length of a time block (unit of analysis)
- Run length for each movie

De Munt Theaters, The Netherlands, has provided the required data for January 10, 2002 to January 16, 2002. The data includes all required information about the number of movies available for running, run-length of the movies, number of screens, capacity of individual screens and the movie schedule (demand data points). The slot-by-slot demand matrix is generated by extrapolating and interpolating the demand points (obtained from the schedule), which has been described earlier.

Definition of variables

S	Total number of screen in a theater,
T	Length of planning horizon (total number of time blocks),
N	Number of candidate movies,
Open	Time the theater opens for showing movies,
Close	Time the theater closes for showing movies,

$NumBlocks$	$\frac{Close - Open}{Block}$
rt_{1j}	Number of blocks for run-time of movie j,
rt_j	Extended number of blocks for run time of movie j,
C	Cleaning time

where,

$$rt_j = rt_{1j} + C$$

Block length chosen is of 10 minutes. Basically, we divide the day into equal sized blocks. We assume, after we show a movie, that it takes an integral number of blocks to get the screen ready to be used again and this multiple is independent of the movie shown. This multiple is what we add to rt_{1j} to get rt_j.

We now introduce the critical new binary variable $s_{j,l,t}$. This is one if on Screen l, we start showing movie j at block t and it runs for time rt_{1j} and zero otherwise.

4.5.1 Basic Model (Model 1):

The basic model poses only the elementary movie-scheduling constraints on micro-scheduling problem. None of the complete managerial constraints is included in this model. The model includes the following constraints.

a) At each time slot, at most one movie is running on each screen.

b) At each time slot, each movie should run on at most one screen.

c) No movie should open before the opening time that is 10:30 hrs and all the movies shall close before or at the closing time, that is 00:10 hrs.

d) No movie should start during break period.

Objective Function

The objective of the problem is to maximize the revenue earned from all movies scheduled. This can be formulated as:

Objective

Maximize total revenue from all movies scheduled

$$\sum_{j=1}^{N}\sum_{l=1}^{S}\sum_{t=1}^{T}\{R[j,l,t]*s[j,l,t]\}$$

j in N (MOVIE), l in S (SCREEN), t in T (TIME)

where,

$R\,[j,\,l,\,t]$ = Revenue of the j movie for t time-slot and when screened on l screen.

$R\,[j,\,l,\,t] = (ATP + Pop) * \min\,\{Cap[l],\;\; Demand\,[j,\,t]\}$

$s\,[j,\,l,\,t] = 1$, if on screen l we start movie j at t; 0, otherwise

$s\,[j,\,l,\,t]$ = Binary variable

ATP = Average ticket price

Pop = Concession revenues per visitor

$Cap[l]$ = Capacity of screen l

$Demand\,[j,\,t]$ = Demand of movie j at t

Break start = starting slot of the break period

Break end = ending slot of the break period

Subject to constraints:

1. Scheduling variable $s[j,l,t]$ is binary

$\forall l, \forall j, \forall t \qquad\qquad s[j,l,t] \in \{0,1\}$

2. Cannot schedule a movie before open (10:30) and after close (00:10)

$\forall l, \forall j,\;\; rt_j$ = Extended number of blocks for run time of movie j

$s[j,l,t] = 0$, if $t < Open$, and $t + rt_{lj} + c > Close$

3. For any screen l, at most one movie is being shown at time T

$$\forall l,\ \forall T \qquad \sum_{j=1}^{N} \sum_{t=T-rt_j+1}^{T} s[j,l,t] \leq 1$$

4. For any movie j, it is only being shown on one screen at a given time T

$$\forall j,\ \forall T \qquad \sum_{l=1}^{S} \sum_{t=T-rt_j+1}^{T} s[j,l,t] \leq 1$$

5. No movie should be screened during the break period that is in the period from slot 37 to 40.

$$\forall j, \forall l, \forall t \qquad s[j,l,t] = 0, \text{ if } \quad 37 <= t <= 40$$

The following four extensions were identified for the basic micro-scheduling problem:

a) Basic High-low

b) Basic Twenty minutes

c) Basic Hopping

d) Basic Hopping High-low

The above four models were developed by adding the managerial constraints to the basic model. We will discuss the four extensions one by one in the following sub-sections.

4.5.2 Model 2 (High-Low Extension):

The basic model when mixed with the managerial high-low constraint gives rise to the basic high-low variant. The new high-low constraint is described as follows:

High-Low constraint: During the evening session, for each time slot, no more than one movie should start on the set of Screens 1 to 8 and apparently, at the set of Screens 9 to 13. (The total number of screens in the DeMunt Theater is 13). Screens 1 to 8 are called the low-capacity screens and the other group of screens is called high-capacity screens. In other words, the constraint can be stated as; at

most one movie should start on the high-capacity screens and at most one on the low-capacity screen in one slot.

This managerial constraint is added to avoid the crowd not to exceed the floor capacity. Because by starting two movies on different screens on the same floor simultaneously, crowd can exceed the capacity of the floor.

The mathematical formulation for the above variant is as follows:

Objective

This is the same as described in model 1.

Subject to constraints:

There are two new constraints of high-low screen other than the basic constraints of the first model.

1. At most one movie starts in the set Low screen (l in Screen # 1 to 8) at a given point of time during the evening session.

$$\forall t > 46 \qquad \sum_{j}\sum_{l} s[j,l,t] \le 1$$

2. At most one movie starts in the set High screen (l in Screen # 9 to 13) a given point of time during evening session.

$$\forall t > 46 \qquad \sum_{j}\sum_{l} s[j,l,t] \le 1$$

4.5.3 Model 3 (Twenty Minutes Extension):

This extension of the basic model consists of basic constraints of first model and the managerial constraint that a new movie must start every twenty minutes. The new managerial constraint of twenty minute is:

Twenty minutes constraint: At least one movie should start every twenty minutes. This constraint makes sure that a visitor, once he/she enters the

multiplex, never has to wait for more than twenty minutes to watch a movie. The IP formulation for the variant is as follows:

Objective

This is same as described in model 1.

Subject to constraints:

There is only one new managerial constraint other than the basic constraints of first model. The new constraint is:

Twenty Minutes constraint: The duration of the period for which the constraint is to be implemented is t_1 to t_2 where t_1 is the starting slot and the ending slot is t_2.

$$\forall t \in (t_1, t_2)$$

$$\sum_{t=t_1}^{t_2} \sum_{l=1}^{S} (s[j,l,t] + s[j,l,t+1] + s[j,l,t+2]) >= 1, \ \forall j$$

In the current problem, value of t_1 and t_2 are respectively 1 and 67. Value of t_2 is 67 because after 67 slot it would not be possible to follow the 20-minute constraint.

4.5.4 Model 4 (Hopping Extension):

This model includes the hopping constraint along with the basic constraint set. The basic constraint set is same as in first model. The hopping constraint is:

Hopping Constraint: A movie should not change screen during the morning and evening sessions and should stay on the screen once allocated. Changing screens is allowed only between the morning and evening sessions. When a movie changes screens within morning and evening sessions then it is termed as a hop. A penalty is associated with the screen switching and is called the hop-cost.

The model is as follows:

Objective

Maximize total revenue from all movies scheduled

$$\sum_{j=1}^{N}\sum_{l=1}^{S}\sum_{t=1}^{T}\{R[j,l,t]*s[j,l,t]\}\text{-}hopcost*\{\sum_{j=1}^{N}\sum_{l=1}^{S}b[j,l]\}$$

$$+hopcost*\{\sum_{j=1}^{N}c[j]\}\text{-}hopcost*\{\sum_{j=1}^{N}\sum_{l=1}^{S}mornb[j,l]\}+hopcost*\{\sum_{j=1}^{N}mornc[j]\}$$

j in N (MOVIE), l in S (SCREEN), t in T (TIME)

where,

mornb[*j*,*l*] = 1, if on screen l we start movie j in morning; 0, otherwise

b[*j*,*l*] = 1, if on screen l we start movie in evening; 0, otherwise

mornc[*j*] = 1, if movie j is played in morning; 0, otherwise

c[*j*] = 1, if movie j is played in evening; 0, otherwise

hopcost = penalty cost, taken as 300

Subject to constraints

There are eight new managerial constraints. First four for morning session and second four for evening session.

New constraints (1, 2, 3 and 4) allow hopping, with cost (Morning)

1. $\forall j, \forall l$ $\quad 5*mornb[j,l] - \sum_{t=1}^{T\_afternoon-rt_1+1} s[j,l,t] >= 0$

2. $\forall j, \forall l$ $\quad -mornb[j,l] + \sum_{t=1}^{T\_afternoon-rt_1+1} s[j,l,t] >= 0$

3. $\forall j$ $\quad -mornc[j] + \sum_{l=1}^{S} \mathrm{mornb}[j,l] >= 0$

4. $\forall j$ $\quad 5*mornc[j] - \sum_{l=1}^{S} \mathrm{mornb}[j,l] >= 0$

where,

$$T\_afternoon = 45$$

New constraints (1, 2, 3, and 4) allow screen hopping, with cost (Evening)

1. $\forall j, \forall l$ $$5*b[j,l] - \sum_{t=T\_evestart}^{T-rt_1+1} s[j,l,t] >= 0$$

2. $\forall j, \forall l$ $$-b[j,l] + \sum_{t=T\_evestart}^{T-rt_1+1} s[j,l,t] >= 0$$

3. $\forall j$ $$-c[j] + \sum_{l=1}^{S} b[j,l] >= 0$$

4. $\forall j$ $$5*c[j] - \sum_{l=1}^{S} b[j,l] >= 0$$

where,

$$T\_evestart = 46$$

4.5.5 Model 5 (Hopping High Low extension):

The basic hop high low extension is a combination of two managerial constraints, namely, the high-low constraint and the hop constraint with the basic constraint.

The two formulations have been explained separately earlier.

CHAPTER 5

EMPIRICAL ANALYSIS

For empirical analysis, we have chosen data from January 10, 2002 to January 16, 2002 of De Munt Theater. The numbers of movies available for playing varies from 18 to 20. We have some data points (actual demand) available for different movies for different days. These different types of movies have different demand pattern over the day. For estimating the demand during different time slots, we have followed some estimation rules, which have been defined earlier.

In the empirical analysis procedure, we generate the optimal schedule by running our mathematical models on AMPL software. Appendix (A) contains the model files for proposed models. We then compare the results of optimal schedule with the actual schedule as played by the exhibitor. Comparison of different schedules can be done based on several factors. The major factors considered in our analysis are net revenue, occupied time slots, screen utilization, average time slots per movie, and percentage improvement in revenue in comparison to actual revenue.

For analysis purpose, the whole day has been divided into 83 equal slots of 10 minutes each. There are total 13 screens available. In this way, there are total 1079 slots available. Occupied time slots are the number of slots utilized by a schedule out of 1079 slots.

Screen utilization is the percentage form of occupied time slots. Screen utilization is calculated by the following formula.

Screen utilization = (Total Occupied Slots/Total Number of Slots Available)*100

Average time slots per movie are the average number of time slots occupied by a movie. This is determined by the following formula:

Average Time slots/movie

= (Total No. of occupied slots/Total no. of movies played)

Percentage improvement in revenue in comparison to actual is calculated by the following formùla.

Percentage Improvement

= {(Optimal Revenue- Actual Revenue)/ (Actual Revenue)}*100

Sensitivity Analysis

Through sensitivity analysis, we try to check the robustness of results of models. Towards this end, we change different parameters of our model and examine the effect of the change on the results. For testing of models, 48 test problems were generated using the data of January 10, 2002. The base data of January 10, 2002 has been shown in Appendix (B). These problems have been categorized into five groups based on inputs provided. The categories are:

- Number of movies
- Capacity of screens
- Demand of movies
- Run length of movies
- Number of screens

In the **first group**, eight problems have been generated to test the influence of the number of movies on the solution. In this group, better and worse movie have been selected based on revenue. Movies 3 and 11 were found better and movies 15 and 17 were found worse,

out of the 20 movies for January 10, 2002. The first four problems represent a scenario ... which the movies with highest expected revenue are added, or the movies with the least revenues removed from the list. The rest four problems represents a situation where by there are more movies with lesser than normal revenues.

These eight problems are:

Problems	Number of movies
Add two good movies	20
Add one good movie	19
Subtract two worst movies	16
Subtract one worst movie	17
Add two worst movies	20
Add one worst movie	19
Subtract two good movies	16
Subtract one good movie	17

Table 5.1: First Group of Test Problems

In this way, eight alternative scenarios/seasons were generated for testing.

In the **Second group**, eight different problems have been generated. These are:

Problems	Capacity of Screens
1	Capacity of all screens increased by 5 %
2	Capacity of all screens increased by 10 %
3	Capacity of all screens increased by 15 %
4	Capacity of all screens increased by 20 %
5	Capacity of all screens increased by 5 %
6	Capacity of all screens increased by 10 %
7	Capacity of all screens increased by 15 %
8	Capacity of all screens increased by 20 %

Table 5.2: Second Group of Test Problems

In **third group**, sixteen problems have been generated. These problems can be categorized under the two heads-the optimistic and pessimistic scenario. The numbers between 5 and 10 are generated following a uniform distribution. The following example illustrates the approach. Suppose there is a demand matrix for 3 time slots and 3 movies. The original demand matrix is say as shown.

20	25	15
42	15	8
0	22	30

Table 5.3: Sample Demand Matrix

Now we generate a matrix of random numbers between 5 and 10 of same size as that of the demand matrix that is 3 x 3 in this case.

5	10	8
9	7	7
10	6	5

Table 5.4: Random Number Matrix

In order to generate the demand matrix for the optimistic scenario and corresponding to the base demand matrix, select an element from the base demand matrix and increase it by a "number" percent, where "number" is the element from the random number matrix. For example, the [1, 1] Demand matrix element is 20. The corresponding number in the random number matrix is 5. In the test problem demand matrix the [1, 1] element will be five percent of twenty added to twenty. Numerically,

[X, Y] of test problem demand matrix

= [X, Y] of base demand matrix (1+ [X, Y] of the random matrix/100)

For pessimistic scenario, everything remains the same except that instead of increasing the base demand matrix values are decreased and that explains the negative sign in the following equation. Numerically,

[X, Y] of test problem demand matrix

= [X, Y] of base demand matrix (1- [X, Y] of the random matrix/100)

For optimistic case, the following table will give the demand matrix of test problem.

21	28	16
46	16	9
0	23	32

Table 5.5: Test Problem Demand Matrix

In the **fourth group**, eight different problems have been generated. These are:

Problems	**Run Length of Movies**
1	Run length of movies increased by 5 %
2	Run length of movies increased by 10 %
3	Run length of movies increased by 15 %
4	Run length of movies increased by 20 %
5	Run length of movies decreased by 5 %
6	Run length of movies decreased by 10 %
7	Run length of movies decreased by 15 %
8	Run length of movies decreased by 20 %

Table 5.6: Fourth Group of Test Problems

In the **fifth group,** eight different problems have been generated. The base data has the highest capacity of 384 and 340; similarly, the least capacity ones have space for 90 and 96 people. The variations of number of screens considered are as follows:

Problems	Number of Screens
Add two highest capacity screens	15
Add one highest capacity screen	14
Remove one least capacity screen	12
Remove two least capacity screens	11
Add two least capacity screens	15
Add one least capacity screen	14
Remove two highest capacity screens	12
Remove one highest capacity screen	11

Table 5.7: Fifth Group of Test Problems

The sensitivity analysis involves the comparisons of the actual and optimal solution for the 48 test problems generated. The optimal solution refers to the solution obtained by running our mathematical models. The actual solutions are not available for the test problems for comparisons. The actual solutions for test problems are obtained using the following approaches:

For the 16 test problems (third group), which have been generated by changing the demand of movies of the base data, the actual revenue are calculated using the actual schedule for the base data, (Base data refers to January 10, 2002 obtained from DeMunt). The revenue for the problem is calculated using the new demand matrix instead of the original January 10, 2002 demand matrix and assuming that the movie allocation for the day remains the same.

For the rest of the test problems it is assumed that ratio of the optimal schedule and the actual schedule remains the same. Hence, the actual revenue for the rest of the test problems is calculated using the following formula:

$$\text{actrev} = \frac{\text{actrev\_basedata}}{\text{optrev\_basedata}} * \text{optrev}$$

where,

actrev = actual revenue for the test problem

optrev = optimal revenue for the test problem

actrev_basedata = actual revenue for the base data

optrev_basedata = optimal revenue for the base data

CHAPTER 6

RESULTS AND DISCUSSION

6.1 Empirical Analysis

For empirical analysis, we have compared the results of optimal schedules generated by our mathematical models with actual schedules generated by exhibitor based on different parameters for the data from January 10, 2002 to January 16, 2002.

The following three main factors have been compared for all five models: net revenue, percentage improvement in revenue, and total numbers of visitors. Some additional factors, such as, occupied time slots, average time slots per movie, and screen utilization have also been compared for all five models.

The results of empirical analysis are shown in Tables 6 and 7.

Comparison Tables:

Table 6: Performance of Proposed Models Based on Main Factors

Date	Factors	Actual Schedule	Optimal Schedule (Basic)	Optimal Schedule (Basic+20 mins)	Optimal Schedule (Basic+Hi-Lo)	Optimal Schedule (Basic+Hopping)	Optimal Schedule (Basic+Hopping+Hi-Lo)
10th Jan	Net Revenue	46750 (-)	64005 (36.9)	65005 (36.9)	63354 (35.5)	63070 (34.9)	52037 (11.3)
	Total Visitors	2750	3765	3765	3727	3710	3061
11th Jan	Net Revenue	67303 (-)	90151 (33.94)	90151 (33.94)	89471 (32.93)	86156 (28.01)	78285 (16.31)
	Total Visitors	3959	5303	5303	5263	5068	4605
12th Jan	Net Revenue	9371 (-)	124151 (32.46)	124015 (32.32)	122638 (30.85)	119170 (27.15)	116824 (24.65)
	Total Visitors	5513	7303	7295	7214	7010	6872
13th Jan	Net Revenue	90882 (-)	121584 (33.78)	121456 (33.64)	120261 (32.32)	118830 (30.75)	113220 (24.57)
	Total Visitors	5346	7152	7145	7074	6990	6660
14th Jan	Net Revenue	40171 (-)	55148 (37.28)	55148 (37.28)	54676(36.1)	52819 (31.48)	51544 (28.31)
	Total Visitors	2363	3244	3244	3216	3107	3032
15th Jan	Net Revenue	38131 (-)	51408 (34.81)	51391 (34.77)	50723 (33.02)	48994 (28.48)	43746 (14.72)
	Total Visitors	2243	3024	3023	2984	2882	2573
16th Jan	Net Revenue	46342 (-)	63376 (36.75)	63325(36.64)	62851 (35.62)	55896 (20.61)	53805 (16.1)
	Total Visitors	2726	3728	3725	3697	3288	3165

*** Percentage improvement in revenue over actual revenue is shown in parentheses.**

Table 7: Performance of Proposed Models Based on Capacity Utilization

Data	Factors	Actual Schedule	Optimal Schedule (Basic)	Optimal Schedule (Basic+20 min)	Optimal Schedule (Basic+Hi-Lo)	Optimal Schedule (Basic+Hopping)	Optimal Schedule (Basic+Hopping+Hi-Lo)
10th Jan	Occupied Slots	664 (61.53)	859 (79.51)	894(82.85)	843 (78.12)	812 (75.25)	742 (69.41)
	Avg Time Slots/Movie	36.88	47.72	49.66	46.83	45.11	43.64
11th Jan	Occupied Slots	688 (63.76)	864 (80.07)	895 (82.94)	849 (78.68)	876 (81.94)	760 (71.09)
	Avg Time Slots/Movie	38.22	48	49.72	47.16	48.66	42.22
12th Jan	Occupied Slots	645 (59.77)	816 (75.62)	867 (80.35)	801 (74.23)	842 (78.76)	869 (81.29)
	Avg Time Slots/Movie	32.25	40.8	43.35	40.05	42.1	43.45
13th Jan	Occupied Slots	704 (65.24)	841 (77.94)	897 (83.13)	836 (77.47)	843 (78.85)	826 (77.26)
	Avg Time Slots/Movie	35.2	42.05	44.85	41.8	44.36	41.3
14th Jan	Occupied Slots	652 (60.42)	874 (81)	896 (83.03)	856 (79.33)	872 (81.57)	739 (69.13)
	Avg Time Slots/Movie	36.22	48.55	49.77	47.55	48.44	43.47
15th Jan	Occupied Slots	663 (61.44)	832 (77.1)	891 (82.57)	829 (76.83)	792 (74.08)	824 (77.08)
	Avg Time Slots/Movie	36.83	46.22	49.5	46.05	49.5	48.47
16th Jan	Occupied Slots	671 (62.18)	856 (79.33)	886 (82.11)	855 (79.24)	787 (73.62)	870 (81.38)
	Avg Time Slots/Movie	33.55	42.8	44.3	42.75	39.35	45.78

* Screen utilization is shown in parentheses

* Avg = Average

Discussion:

From the results shown in comparison tables, it is clear that the schedules generated by the proposed models are substantially better than the actual schedule made by exhibitors.

We have taken 7 days data from January 10, 2002 to January 16, 2002 for testing of our models. The number of movies available for playing varies between 18 and 20. There are 13 screens of different capacity available in the theatre.

The increase in net revenue varies from 32 to 38% for different data sets. The maximum improvement in net revenue is achieved from basic model that gradually reduces by adding managerial constraints in basic model, because the complexity of model increases. However, in spite of the increase in complexity revenue is much better in case of proposed models than the actual schedule. The reasons for lower revenue in actual schedule may be that the manager in actual situation might be honouring some constraints, which we may not have accounted for.

One more interesting thing comes into picture from results that when we add 20 minutes constraint into basic model, then sometimes the net revenue does not change from the value of basic model for 18 numbers of movies. It may be because of data sets, which are being used for model formulation. However, it decreases by some amount in the case of 20 numbers of movies. Also occupied number of slots and screen utilization increases significantly in the optimal schedules. Here one thing is to see that in the case of **Basic + 20 minutes** schedule the occupied number of slots and the screen utilization is maximum. This is because in this some movies having zero demand have been scheduled by model due to constraint of 20 minute.

Our results indicate that by using the mathematical models for scheduling the movies in a multiplex theater we can improve the net revenue up to a large extent. Although by adding more managerial constraints results in making model more complex to make it more realistic, net revenue will be reduced by some amount, it is likely to be better than net revenue given by actual schedule. In addition, screen utilization, occupied time slots, average number of time slots per movie will also increase and resource utilization will be better.

6.2 Sensitivity Analysis

As explained in the earlier chapter, we generated 48 test problems for the data of January 10, 2002. The last two models take too long time; therefore, these have been tested for only first 16 problems. The results of sensitivity analysis are shown in Tables 8.1 and 8.2.

Table 8.1: Comparison of Proposed Models with Actual on the Basis of Revenue for First 16 Problems

10th Jan 2002	Factors	Actual Schedule	Optimal Schedule Basic	Optimal Schedule (Basic+20 mins)	Optimal Schedule (Basic+Hi-Lo)	Optimal Schedule (Basic+Hopping)	Optimal Schedule (Basic+Hopping+Hi-Lo)
Problem No. 1	Net Revenue	59304 (-)	81192 (36.90)	81192 (36.90)	80699 (36.07)	78581 (32.5)	72845 (22.83)
Problem No. 2	Net Revenue	45682 (-)	62543 (36.90)	62543 (36.90)	62492 (36.79)	60595 (32.64)	60236 (31.85)
Problem No. 3	Net Revenue	47767 (-)	65399 (36.90)	65399 (36.90)	65348 (36.80)	63324 (32.56)	62263 (30.34)
Problem No. 4	Net Revenue	24722 (-)	33847 (36.91)	33847 (36.91)	33745 (36.49)	32767 (32.54)	28561 (15.52)
Problem No. 5	Net Revenue	47060 (-)	64430 (36.91)	64430 (36.91)	64379 (36.80)	62171 (32.11)	56368 (20.00)
Problem No. 6	Net Revenue	47060 (-)	64430 (36.91)	64430 (36.91)	64379 (36.80)	62495 (32.80)	62379 (32.55)
Problem No. 7	Net Revenue	46513 (-)	63682 (36.91)	63682 (36.91)	63631 (36.80)	61702 (32.65)	57134 (22.83)
Problem No. 8	Net Revenue	46153 (-)	63189 (36.91)	63189 (36.91)	63138 (36.80)	61157 (32.50)	61107 (32.40)
Problem No. 9	Net Revenue	51517 (-)	70533 (36.91)	70533 (36.91)	70499 (36.84)	68292 (32.56)	63353 (23.00)
Problem No. 10	Net Revenue	42726 (-)	58497 (36.91)	58497 (36.91)	58463 (36.83)	56665 (32.62)	55630 (30.20)
Problem No. 11	Net Revenue	42043 (-)	57562 (36.91)	57562 (36.91)	57545 (36.87)	55722 (32.53)	55322 (28.34)
Problem No. 12	Net Revenue	46749 (-)	64005 (36.90)	64005 (36.90)	63954 (36.80)	62048 (32.62)	61999 (32.72)
Problem No. 13	Net Revenue	47780 (-)	65416 (36.90)	65416 (36.90)	65382 (36.83)	58256 (21.92)	58256 (21.92)
Problem No. 14	Net Revenue	46017 (-)	63002 (36.90)	63002 (36.90)	62985 (36.87)	61024 (27.33)	60816 (27.14)
Problem No. 15	Net Revenue	47780 (-)	65416 (36.90)	65416 (36.90)	65263 (36.59)	60839 (27.33)	60750 (27.14)
Problem No. 16	Net Revenue	44030 (-)	60282 (36.90)	60282 (36.90)	60095 (36.48)	58382 (32.59)	58250 (32.29)

* Percentage improvement in revenue over actual revenue is shown in parentheses

Table 8.2 (a): Comparison of First Three Models with Actual Based on Revenue for *Rest 32 Problems*

10th Jan 2002	Factors	Actual Schedule	Optimal Schedule (Basic)	Optimal Schedule (Basic+20 mins)	Optimal Schedule (Basic+Hi-Lo)
Problem No. 17	Net Revenue	64075 (-)	87720 (36.90)	87652 (37.12)	87108 (35.94)
Problem No. 18	Net Revenue	46265 (-)	63342 (36.90)	63342 (36.91)	63291 (36.80)
Problem No. 19	Net Revenue	47258 (-)	64702 (36.90)	64685 (36.87)	64668 (36.84)
Problem No. 20	Net Revenue	32085 (-)	43928 (36.91)	43928 (36.91)	43877 (36.75)
Problem No. 21	Net Revenue	47060 (-)	64430 (36.91)	64430 (36.91)	64379 (36.80)
Problem No. 22	Net Revenue	47060 (-)	64430 (36.91)	64430 (36.91)	64379 (36.80)
Problem No. 23	Net Revenue	45595 (-)	62424 (36.91)	62424 (36.90)	62288 (36.61)
Problem No. 24	Net Revenue	45148 (-)	61812 (36.91)	61812 (36.90)	61642 (36.53)
Problem No. 25	Net Revenue	51542 (-)	70091 (35.98)	70091 (35.98)	70054 (35.91)
Problem No. 26	Net Revenue	51779 (-)	70448 (36.05)	70431 (36.02)	70363 (35.85)
Problem No. 27	Net Revenue	51754 (-)	70414 (36.05)	70397 (36.02)	70312 (35.85)
Problem No. 28	Net Revenue	51704 (-)	70346 (36.05)	70346 (36.05)	70295 (35.95)
Problem No. 29	Net Revenue	51892 (-)	70601 (36.05)	70584 (36.02)	70584 (36.02)
Problem No. 30	Net Revenue	51879 (-)	70584 (36.05)	70567 (36.02)	70465 (35.82)
Problem No. 31	Net Revenue	51442 (-)	70057 (36.91)	70057 (36.18)	70006 (36.08)
Problem No. 32	Net Revenue	43174 (-)	59143 (36.18)	59143 (36.98)	59126 (36.94)
Problem No. 33	Net Revenue	43298 (-)	59313 (36.45)	59313 (36.98)	59228 (36.79)

* **Percentage improvement in revenue over actual revenue is shown in parentheses**

Table 8.3 (b): Continuation of Table 8.2 (a)

10th Jan 2002	Factors	Actual Schedule	Optimal Schedule (Basic)	Optimal Schedule (Basic+20 mins)	Optimal Schedule (Basic+Hi-Lo)
Problem No. 34	Net Revenue	43547 (-)	59653 (36.91)	59619 (36.90)	59517 (36.67)
Problem No. 35	Net Revenue	43001 (-)	58905 (36.98)	58888 (36.94)	58871 (36.90)
Problem No. 36	Net Revenue	42802 (-)	58633 (36.98)	58633 (36.98)	58616 (36.94)
Problem No. 37	Net Revenue	43224 (-)	59211 (36.98)	59177 (36.90)	59177 (36.67)
Problem No. 38	Net Revenue	42554 (-)	58293 (36.98)	58293 (36.98)	58276 (36.94)
Problem No. 39	Net Revenue	40193 (-)	55029 (36.91)	54978 (36.78)	54978 (36.78)
Problem No. 40	Net Revenue	38554 (-)	52785 (36.91)	52751 (36.82)	52734 (36.77)
Problem No. 41	Net Revenue	36344 (-)	49759 (36.91)	49708 (36.91)	49572 (36.39)
Problem No. 42	Net Revenue	49481 (-)	67745 (36.91)	67626 (36.67)	67490 (36.54)
Problem No. 43	Net Revenue	50946 (-)	69751 (36.91)	69649 (36.71)	69564 (36.54)
Problem No. 44	Net Revenue	53976 (-)	73899 (36.91)	73865 (36.84)	73712 (36.56)
Problem No. 45	Net Revenue	47334 (-)	64804 (36.90)	64804 (36.90)	64753 (36.80)
Problem No. 46	Net Revenue	46227 (-)	61778 (33.64)	61761 (33.60)	61710 (33.49)
Problem No. 47	Net Revenue	47234 (-)	64804 (37.19)	64804 (37.19)	64685 (36.94)
Problem No. 48	Net Revenue	40504 (-)	55454 (36.90)	55386 (36.74)	55284 (36.49)

* **Percentage improvement in revenue over actual revenue is shown in parentheses**

Discussion:

Table 8.1 contains results for 16 test problems. In the table, we have compared the revenue of these 16 problems for all five models with the actual revenue for that particular test problem. For the rest of the 32 test problems, we have compared the revenue for the first three models that is for basic, basic+20 minutes, basic+hi-lo. The results for these 32 test problems have been shown in Table 8.2.

From the results shown in Tables 8.1 and 8.2 it is clear that the revenue generated for each test problem by basic model is much better than the actual revenue for that particular test problem. However, it is also clear from the tables that as the complexity of model increases, the improvement in revenue decreases.

Thus, we find that, our mathematical models are robust in nature and the general direction of the expected results is maintained.

6.3 Managerial Implications

Micro-scheduling is a complex problem and its modeling itself is a contribution towards management. The contribution towards managerial help can come in two ways-better scheduling and faster solution. Results from current research show that the mathematical models presented have potential to provide improvement on both fronts. The models can help the manager adopt a new decision-making style, which is based on an analytical approach.

These models may automate part of the scheduling process and help in saving management time.

6.4 Limitations and Scope for Future Research

We now discuss some limitations of the current approach, which provide potential future research ideas. When we increase the complexity of models by adding some managerial constraints to make them more realistic, then time to solve the models increases substantially and in most cases, it does not converge completely. Therefore, time is the main constraint in complex situations and as the complexity of models will increase, time taken by the program will increase.

There are some managerial constraints, which could be added to models to increase the complexity such as floor capacity, multiple copies of movies, kids movies, must-play/committed movies, and so on. After adding these constraints to models, complexity analysis of the optimization models would provide useful insights into the efficiency aspects of the problem.

References

Swami, Sanjeev, Eunkyu Lee, and Charles B. Weinberg (1998), "Optimal Channel Contracts for Marketing Perishable Products," Working Paper, University of British Columbia.

Swami, Sanjeev, Jehoshua Eliashberg, and Charles B. Weinberg (1999), "SilverScreener: A Modeling Approach to Movie Screens Management," *Marketing Science*, 18 (3), pp. 352-372.

Swami, Sanjeev, Jehoshua Eliashberg, Charles B. Weinberg and Berend Wierenga (2001), "Implementing and Evaluating SILVERSCREENER: A Marketing Management Support System for Movie Exhibitors," forthcoming *Interfaces*: Special issue on Marketing Engineering.

Variety, The International Entertainment Weekly.

Corstjens, M. and Doyle, P. (1983), "A Model for Optimizing Retail Space Allocations," *Management Science,* 27 (July), 822-833.

Weinberg, Charles B. (1986), "ARTS PLAN: Implementation, Evolution and Usage," *Marketing Science*, 5 (2), 143--158.

Zufryden, Fred S. (1996), "Linking Advertising to Box Office Performance of New Film Releases-A Marketing Planning Model," *Journal of Advertising Research*, (July-August), 29-41.

Sawhney, Mohanbir S. and Jehoshua Eliashberg (1996), "A Parsimonious Model for Forecasting Gross Box Office Revenues of Motion Pictures," *Marketing Science*, 15 (2), 113-131.

Acquilano, N. J., and R. B. Chase (1991), *Fundamentals of Operations Management*, Irwin, Homewood, IL.

Austin, Bruce A. and Thomas F. Gordon (1987), "Movie Genres: Toward a Conceptualized Modeland Standardized Definition," in *Current Research in Film: Audiences, Economics and the Law, Vol. 4*, B. A. Austin, ed., Norwood, NJ: Ablex Publishing Co.

Bultez, Alain and Phillippe Naert (1988), "S.H.A.R.P.: Shelf Allocation for Retailers' Profit," *Marketing Science*, 7, 3 (Summer), 211--231.

Eliashberg, Jehoshua and Steven M. Shugan (1997), "Film Critics: Influencers or Predictors?"*Journal of Marketing*, 61, 2 (April), 68-78.

Dodds, John C. and Morris B. Holbrook (1988), “What's an Oscar Worth? An Empirical Estimation of the Effect of Nominations and Awards on Movie Distribution and Revenues,”in *Current Research in Film: Audiences, Economics and the Law, Vol. 4*, B. A. Austin, ed., Norwood, NJ: Ablex Publishing Co.

Eliashberg, Jehoshua; Jonker, Jedid-Jah; Sawhney, Mohanbir S.; and Wierenga, Berend (2000), “MOVIEMOD: An implementable decision support system for pre-release market evaluation of motion pictures” *Marketing Science*, Volume 19, Number. 3, pp. 226-243.

Eliashberg, Jehoshua and Mohanbir S. Sawhney (1994), “Modeling Goes to Hollywood: Predicting Individual Differences in Movie Enjoyment,” *Management Science*, 40 (September),1151-1173.

Jain, Karuna and Edward A. Silver (1994), “Lot Sizing for a Product Subject to Obsolescence or Perishability,” *European Journal of Operational Research*, 75 (2), 287--295.

Jedidi, Kamel, Robert E. Krider, and Charles B. Weinberg (1998), “Clustering at the Movies,” *Marketing Letters,* 9 (4), 393-405.

Jones, Morgan and Christopher J. Ritz (1991), “Incorporating Distribution into New Product Diffusion Models,” *International Journal of Research in Marketing*, 8, 91-112.

Krider, Robert E. and Charles B. Weinberg (1998), “Competitive Dynamics and the Introduction of New Products: The Motion Picture Timing Game,” *Journal of Marketing Research*, 35, 1 (February), 1-15.

Lehmann, Donald R. and Charles B. Weinberg (2000), “Sales Via Sequential Distribution Channels: An Application to Movie Audiences,” *Journal of Marketing*, 64 (3), 13-33.

Lodish, Leonard M. (1971), “CALLPLAN: An Interactive Salesman's Call Planning System,” *Management Science*, 18, 4(2), 25--40.

Mahajan, Vijay, Eitan Muller, and Roger Kerin (1984), “Introduction Strategy for New Products with Positive and Negative Word-of-Mouth,” *Management Science*, 30 (December), 1389-1404.

Prasad, Ashutosh, Vijay Mahajan, and Bart J. Bronnenberg (1998), “Product Entry Timing in Dual Distribution Channels: The Case of the Movie Industry,” Working Paper, University of Texas at Austin.

Radas, Sonja and Steven M. Shugan (1998), “Seasonal Marketing and Timing Introductions,” *Journal of Marketing Research*, 35 (August), 296-315.

Rangaswamy, Arvind (1993), "Marketing Decision Models: From Linear Programs to Knowledge-based Systems," *Handbooks in OR & MS*, J. Eliashberg and G. L. Lilien (eds.), Elsevier Science Publishers B.V., The Netherlands, 733--771.

Saxena, Sudhir (2000), "Implimentation of a Marketing Decision Support System for Motion Picture Retailing," Thesis Report, M.Tech., Indian Institute of Technology, Kanpur, India.

Smith, Sharon, P. and V. Kerry Smith (1986), "Successful Movies: A Preliminary Empirical Analysis," *Applied Economics*, 18, 501-507.

APPENDIX-A (1)

Ampl Model File (Basic):

```
param n > 0;                                  #number of movies
param T > 0;                                  #number of slots
param S > 0;                                  #number of screens
param open >0;
param close >0;
param C >0;                                    #cleaning time
set MOVIE := 1 .. n;                           #set of movies
set SCREEN := 1 .. S;                          #set of screens
set TIME := 1 .. T;                           #set of time
param rt1{MOVIE}>0;
param rt{j in MOVIE}:= rt1[j]+C;
param capacity {SCREEN} >0;
param ATP >0;
param POP >0;
param demand {TIME,MOVIE} >=0;
param revenue {j in MOVIE, l in SCREEN, t in TIME}:=
(ATP+POP)*min(capacity[l],demand[t,j]);
var s {j in MOVIE, l in SCREEN, t in TIME} binary;
maximize Total_Revenue:
sum {j in MOVIE, l in SCREEN, t in TIME} revenue[j,l,t]* s[j,l,t];
subject to C1 {j in MOVIE, l in SCREEN, t in TIME}:
        0 <= s[j,l,t] <= 1;
subject to
C2 {j in MOVIE, l in SCREEN, t in TIME: t < open or t+rt1[j] > close}:
s[j,l,t]=0;
subject to
C3 {l in SCREEN, T in TIME}:
sum {j in MOVIE, t in T-rt[j]+1 .. T: t>0} s[j,l,t] <= 1;
```

```
subject to C4 {j in MOVIE, T in TIME}:
sum {l in SCREEN, t in T-rt[j]+1 .. T: t>0} s[j,l,t] <= 1;
subject to
C5 {t in TIME: 37 <= t <= 40}:
sum {j in MOVIE, l in SCREEN} s[j,l,t]=0;
```

APPENDIX-A (2)

Ampl Model File (Basic+20 Minutes):

```
param n > 0;                                        #number of movies
param T > 0;                                        #number of slots
param S > 0;                                        #number of screens
param open >0;
param close >0;
param C >0;                                         #cleaning time
set MOVIE := 1 .. n;                                #set of movies
set SCREEN := 1 .. S;                               #set of screens
set TIME := 1 .. T;                                 #set of time
param rt1 {MOVIE}>0;
param rt{j in MOVIE}:= rt1[j]+C;
param capacity {SCREEN} >0;
param ATP >0;
param POP >0;
param demand {TIME, MOVIE} >=0;
param T_afternoon > 0;
param T_evestart > 0;
param T_EVERY20MINS > 0;
set AFTERNOON := 1 .. T_afternoon;
set EVENING:=  T_evestart .. T;
param revenue {j in MOVIE, l in SCREEN, t in TIME}:=
(ATP+POP)*min (capacity[l], demand [t,j]);
var s {j in MOVIE, l in SCREEN, t in TIME} binary;
maximize Total_Revenue:
sum {j in MOVIE, l in SCREEN, t in TIME} revenue [j,l,t]* s [j,l,t];
subject to C1 {j in MOVIE, l in SCREEN, t in TIME}:
        0 <= s [j,l,t] <= 1;
subject to
```

```
C2 {j in MOVIE, l in SCREEN, t in TIME: t < open or t+rt1[j] > close}:
s [j,l,t]=0;
subject to
C3 {l in SCREEN, T in TIME}:
sum {j in MOVIE, t in T-rt[j]+1 .. T: t>0} s [j,l,t] <= 1;
subject to C4 {j in MOVIE, T in TIME}:
sum {l in SCREEN, t in T-rt[j]+1 .. T: t>0} s [j,l,t] <= 1;
subject to
C5 {t in TIME: 37 <= t <= 40}:
sum {j in MOVIE, l in SCREEN} s [j,l,t]=0;
subject to
C6 {t in EVENING: T_evestart-5 <= t <= T_EVERY20MINS}:
sum {j in MOVIE, l in SCREEN} (s[j,l,t] + s[j,l,t+1] + s[j,l,t+2]) >= 1;
subject to
C7 {t in TIME : 1 <= t <= T_afternoon -11}:
sum {j in MOVIE, l in SCREEN} (s[j,l,t] + s[j,l,t+1] + s[j,l,t+2]) >= 1;
```

APPENDIX-A (3)

Ampl Model File (Basic+Hi-Lo):

```
param n > 0;                                    #number of movies
param T > 0;                                    #number of slots
param S > 0;                                    #number of screens
param open >0;
param close >0;
param C >0;                                     #cleaning time
set MOVIE := 1 .. n;                            #set of movies
set SCREEN := 1 .. S;                           #set of screens
set TIME := 1 .. T;                             #set of time
set HI_SCREEN:= 9 .. 13;
set LO_SCREEN:= 1 .. 8;
param rt1{MOVIE}>0;
param rt{j in MOVIE}:= rt1[j]+C;
param capacity {SCREEN} >0;
param ATP >0;
param POP >0;
param demand {TIME,MOVIE} >=0;
param T_afternoon > 0;
param T_evestart > 0;
param revenue {j in MOVIE, l in SCREEN, t in TIME}:=
(ATP+POP)*min (capacity[l], demand[t,j]);
set AFTERNOON := 1 .. T_afternoon;
set EVENING:=  T_evestart .. T;                 # changed to T_evestart from 46
var s {j in MOVIE, l in SCREEN, t in TIME} binary;
maximize Total_Revenue:
sum {j in MOVIE, l in SCREEN, t in TIME} revenue[j,l,t]* s[j,l,t];
```

```
subject to
C1 {j in MOVIE, l in SCREEN, t in TIME}:
        0 <= s[j,l,t] <= 1;

subject to
C2 {j in MOVIE, l in SCREEN, t in TIME: t < open or t+rt1[j] > close}:
s[j,l,t]=0;
subject to
C3 {l in SCREEN, T in TIME}:
sum {j in MOVIE, t in T-rt[j]+1 .. T: t>0} s[j,l,t] <= 1;
subject to
C4 {j in MOVIE, T in TIME}:
sum {l in SCREEN, t in T-rt[j]+1 .. T: t>0} s[j,l,t] <= 1;
subject to
C5 {t in TIME: 37 <= t <= 40}:
sum {j in MOVIE, l in SCREEN} s[j,l,t]=0;
subject to
C6 {t in TIME: t >= T_evestart}:
sum {j in MOVIE, l in LO_SCREEN} s[j,l,t] <= 1;
subject to
C7 {t in TIME: t >= T_evestart}:
sum {j in MOVIE, l in HI_SCREEN} s[j,l,t] <= 1;
```

APPENDIX-A (4)

Ampl Model File (Basic+Hopping):

```
param n > 0;                                          #number of movies
param S > 0;                                          #number of screens
param T > 0;                                          #number of time slots
set SCREEN: = 1 .. S;                                 #set of screens
param C {SCREEN} >0;                                  #cleaning time
param close > 0;
set MOVIE: = 1 .. n;                                  #set of movies
set TIME: = 1 .. T;                                   #set of time slots
param rt1{MOVIE}>0;                                   #run length of movies
param rt{j in MOVIE, l in SCREEN}:= rt1[j]+ C[l];
param capacity {SCREEN} >0;                           #capacity of the screens
param ATP >0;                                         #average ticket price
param POP >0;
param demand {TIME,MOVIE} >=0;                        #demand matrix
param T_afternoon > 0;
param T_evestart > 0;
param hopcost > 0;
param revenue {j in MOVIE, l in SCREEN, t in TIME}:=
(ATP+POP)*min (capacity[l], demand[t,j]);
set AFTERNOON := 1 .. T_afternoon;
set EVENING:= T_evestart .. T;                        # changed to T_evestart from 46
var s {j in MOVIE, l in SCREEN, t in TIME} binary;
var b {j in MOVIE, l in SCREEN} binary;
var c { j in MOVIE} binary;
var mornb {j in MOVIE, l in SCREEN} binary;
var mornc { j in MOVIE} binary;
maximize Total_Revenue:
( sum {j in MOVIE, l in SCREEN, t in TIME} revenue[j,l,t]* s[j,l,t] )
```

```
 - hopcost * ( sum {j in MOVIE, l in SCREEN} b[j,l] )
 + hopcost * ( sum {j in MOVIE} c[j])
 - hopcost * ( sum {j in MOVIE, l in SCREEN} mornb[j,l] )
 + hopcost * ( sum {j in MOVIE} mornc[j] );
subject to
C1 {j in MOVIE, l in SCREEN, t in TIME}:
        0 <= s[j,l,t] <= 1;
#It makes sure we don't start a movie before it opens, or start a movies so late that it is
still running when the theater closes
subject to
C2 {j in MOVIE, l in SCREEN, t in TIME: t+rt1[j]-1 > close}:
s[j,l,t]=0;
# It makes sure that, on each screen L, at most one movie is playing at time T
subject to
C3 {l in SCREEN, T in TIME}:
sum {j in MOVIE, t in T-rt[j,l]+1 .. T: t>0} s[j,l,t] <= 1;
# It makes sure that each movie j, it is on at most one screen at time T
subject to
C4 {j in MOVIE, T in TIME}:
sum {l in SCREEN, t in T-rt[j,l]+1 .. T: t>0} s[j,l,t] <= 1;
# New constraints (C5A, C5B, C5C, C5d) allow screen hopping, with cost (EVE)
subject to
C5a {j in MOVIE, l in SCREEN}:
5* b[j,l]  -  sum {t in T_evestart .. T - rt1[j] + 1} s[j,l,t] >= 0;
subject to
C5b {j in MOVIE, l in SCREEN}:
- b[j,l] + sum {t in T_evestart .. T - rt1[j] + 1} s[j,l,t] >= 0;
subject to
C5c {j in MOVIE}:
- c[j] + sum {l in SCREEN} b[j,l] >= 0;
```

```
subject to
C5d {j in MOVIE}:
5* c[j] - sum {l in SCREEN} b[j,l] >= 0;
#Allow hopping, with cost (MORNING)
C6a {j in MOVIE, l in SCREEN}:
5* mornb[j,l]  -  sum {t in 1 .. T_afternoon - rt1[j] + 1} s[j,l,t] >= 0;
subject to
C6b {j in MOVIE, l in SCREEN}:
- mornb[j,l] + sum {t in 1 .. T_afternoon - rt1[j] + 1} s[j,l,t] >= 0;
subject to
C6c {j in MOVIE}:
- mornc[j] + sum {l in SCREEN} mornb[j,l] >= 0;
subject to
C6d {j in MOVIE}:
5* mornc[j] - sum {l in SCREEN} mornb[j,l] >= 0;
subject to
C7 {t in TIME: 37 <= t <= 40}:
sum {j in MOVIE, l in SCREEN} s[j,l,t]=0;
```

APPENDIX-A (5)

Ampl Model File (Basic+Hopping+Hi-Lo):

```
param n > 0;                                   #number of movies
param S > 0;                                   #number of screens
param T > 0;                                   #number of time slots
set SCREEN: = 1 .. S;                          #set of screens
param C {SCREEN} >0;                           #cleaning time
param close > 0;
set MOVIE: = 1 .. n;                           #set of movies
set TIME: = 1 .. T;                            #set of time slots
param rt1{MOVIE}>0;                            #run length of movies
param rt{j in MOVIE, l in SCREEN}:= rt1[j]+ C[l];
param capacity {SCREEN} >0;                    #capacity of the screens
param ATP >0;                                  #average ticket price
param POP >0;
param demand {TIME,MOVIE} >=0;                 #demand matrix
param T_afternoon > 0;
param T_evestart > 0;
set HI_SCREEN:= 9 .. 13;
set LO_SCREEN:= 1 .. 8;
param hopcost > 0;
param revenue {j in MOVIE, l in SCREEN, t in TIME}:=
(ATP+POP)*min (capacity[l], demand[t,j]);
set AFTERNOON := 1 .. T_afternoon;
set EVENING:= T_evestart .. T;                 # changed to T_evestart from 46
var s {j in MOVIE, l in SCREEN, t in TIME} binary;
var b {j in MOVIE, l in SCREEN} binary;
var c { j in MOVIE} binary;
var mornb {j in MOVIE, l in SCREEN} binary;
var mornc { j in MOVIE} binary;
```

```
maximize Total_Revenue:
( sum {j in MOVIE, l in SCREEN, t in TIME} revenue[j,l,t]* s[j,l,t]  )
  - hopcost * ( sum {j in MOVIE, l in SCREEN} b[j,l] )
  + hopcost * ( sum {j in MOVIE} c[j])
  - hopcost * ( sum {j in MOVIE, l in SCREEN} mornb[j,l] )
  + hopcost * ( sum {j in MOVIE} mornc[j] );
subject to
C1 {j in MOVIE, l in SCREEN, t in TIME}:
 0 <= s[j,l,t] <= 1;
#It makes sure we don't start a movie before it opens, or start a movies so late that it is
still running when the theater closes
subject to
C2 {j in MOVIE, l in SCREEN, t in TIME: t+rt1[j]-1 > close}:
s[j,l,t]=0;
# It makes sure that, on each screen L, at most one movie is playing at time T
subject to
C3 {l in SCREEN, T in TIME}:
sum {j in MOVIE, t in T-rt[j,l]+1 .. T: t>0} s[j,l,t] <= 1;
# It makes sure that each movie j, it is on at most one screen at time T
subject to
C4 {j in MOVIE, T in TIME}:
sum {l in SCREEN, t in T-rt[j,l]+1 .. T: t>0} s[j,l,t] <= 1;
subject to
C5 {t in TIME: t >= T_evestart}:
sum {j in MOVIE, l in LO_SCREEN} s[j,l,t] <= 1;
subject to
C6 {t in TIME: t >= T_evestart}:
sum {j in MOVIE, l in HI_SCREEN} s[j,l,t] <= 1;
# New constraints (C7A, C7B, C7C, C7d) allow screen hopping, with cost (EVE)
subject to
C7a {j in MOVIE, l in SCREEN}:
```

```
5* b[j,l]  -  sum {t in T_evestart .. T - rt1[j] + 1} s[j,l,t] >= 0;
subject to
C7b {j in MOVIE, l in SCREEN}:
- b[j,l] + sum {t in T_evestart .. T - rt1[j] + 1} s[j,l,t] >= 0;
subject to
C7c {j in MOVIE}:
- c[j] + sum {l in SCREEN} b[j,l] >= 0;
subject to
C7d {j in MOVIE}:
5* c[j] - sum {l in SCREEN} b[j,l] >= 0;
#Allow hopping, with cost (MORNING)
C8a {j in MOVIE, l in SCREEN}:
5* mornb[j,l]  -  sum {t in 1 .. T_afternoon - rt1[j] + 1} s[j,l,t] >= 0;
subject to
C8b {j in MOVIE, l in SCREEN}:
- mornb[j,l] + sum {t in 1 .. T_afternoon - rt1[j] + 1} s[j,l,t] >= 0;
subject to
C8c {j in MOVIE}:
- mornc[j] + sum {l in SCREEN} mornb[j,l] >= 0;
subject to
C8d {j in MOVIE}:
5* mornc[j] - sum {l in SCREEN} mornb[j,l] >= 0;
subject to
C9 {t in TIME: 37 <= t <= 40}:
sum {j in MOVIE, l in SCREEN} s[j,l,t]=0;
```

APPENDIX-B
Base Data of January 10, 2002

```
param S:= 13;
param n:= 18;
param T:= 83;
param open:= 1;
param close:= 83;
param C:= 2;
param ATP:= 15;
param POP:= 2;
param T_afternoon:=45;
param T_evestart:=46;

param rt1:=
                1   12
                2   16
                3   12
                4    9
                5   13
                6   10
                7   13
                8   11
                9   18
               10   13
               11   18
               12    9
               13   11
               14   12
               15   16
               16   11
               17    9
               18    9


param capacity:=
                1  222
                2  222
                3  340
                4  113
                5  102
                6  161
                7  163
                8  172
                9  175
               10  177
               11  382
               12   96
               13   90
```

aram demand:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	3	0	62	0	0	0	1	0	37	0	0	0	0	0	31	2	0	6
2	3	2	66	0	0	0	2	0	39	0	0	0	0	0	30	2	0	6
3	3	3	71	0	0	0	2	0	40	0	3	0	0	0	29	2	0	6
4	4	5	75	0	0	0	3	0	41	0	5	0	0	0	28	3	0	6
5	4	6	79	0	0	0	4	0	43	0	8	0	0	0	27	3	0	6
6	4	8	83	0	0	0	4	0	44	0	11	0	0	1	27	4	0	6
7	4	9	87	0	0	0	5	0	45	0	14	0	0	1	26	4	0	6
8	5	10	91	0	0	0	5	0	47	0	17	0	0	1	25	5	0	6
9	5	12	95	0	0	0	6	0	48	0	19	0	0	1	24	5	0	6
10	5	13	99	0	0	0	7	0	50	0	22	0	1	2	23	5	0	6
11	5	15	104	0	0	0	7	0	51	0	25	0	1	2	23	6	1	6
12	6	16	108	0	0	0	8	0	52	0	28	0	1	2	22	6	3	6
13	6	18	112	0	0	0	9	0	54	0	31	1	1	2	21	7	4	6
14	6	19	116	0	0	0	9	0	55	0	34	1	2	2	20	7	5	6
15	6	21	120	0	0	0	10	0	56	0	36	2	2	3	19	8	7	6
16	7	22	124	0	0	0	11	0	58	0	39	2	2	3	19	8	8	6
17	7	24	128	0	0	0	11	0	59	0	42	3	3	3	18	8	9	6
18	7	25	132	0	0	0	12	0	60	0	45	3	3	3	17	9	11	6
19	7	27	136	0	0	0	12	0	62	0	48	3	3	4	16	9	12	6
20	8	28	141	0	0	0	13	0	63	0	50	4	3	4	15	10	13	6
21	8	30	145	0	0	0	14	0	65	0	53	4	4	4	15	10	15	6
22	9	31	149	0	0	0	14	0	66	0	56	5	4	4	14	11	16	6
23	9	33	153	0	0	0	15	0	67	0	59	5	4	5	13	11	18	6
24	9	34	157	0	0	0	16	0	69	0	62	6	4	5	12	11	19	6
25	10	36	161	0	0	0	16	0	70	0	65	6	5	5	11	11	20	6
26	10	37	165	0	0	0	17	0	71	0	67	6	5	5	10	12	22	6
27	11	37	169	0	0	0	17	0	73	0	70	6	5	6	10	12	23	6
28	11	37	174	0	0	0	18	0	74	0	73	6	5	6	9	12	22	6
29	12	38	178	0	0	0	19	0	75	0	76	5	6	6	8	12	21	6
30	12	38	182	0	0	0	19	0	77	0	79	5	6	6	7	13	20	7
31	12	38	186	0	0	0	20	0	78	0	81	5	6	7	6	13	19	7
32	13	38	190	0	0	0	20	0	80	0	84	5	6	7	6	13	18	8
33	13	38	195	0	0	0	20	0	81	0	87	5	6	8	5	13	17	8
34	14	39	201	0	0	0	20	0	82	0	90	5	6	8	4	13	17	9
35	14	39	206	0	0	0	20	0	84	0	93	4	6	8	3	14	16	9
36	15	39	212	0	0	0	21	0	85	0	96	4	6	9	2	14	15	10
37	17	39	217	0	0	0	21	0	89	0	98	4	6	9	2	14	14	10
38	18	39	222	0	0	0	21	0	92	0	101	4	6	10	1	14	13	11
39	20	40	228	0	0	0	21	0	96	0	104	5	7	10	0	14	12	11
40	21	40	233	0	0	0	21	0	99	0	116	6	7	10	0	15	11	12
41	23	40	239	0	0	0	21	0	103	0	127	7	7	11	0	15	10	12
42	24	40	244	0	0	0	21	0	106	0	139	9	7	11	0	15	9	12
43	26	40	249	0	0	0	21	0	110	0	151	10	7	11	0	15	8	13
44	27	41	255	0	0	0	21	0	113	0	162	11	7	12	0	16	7	13
45	29	41	260	0	0	0	22	0	117	0	174	12	7	12	0	16	6	14
46	30	41	266	40	2	35	22	96	120	49	185	13	7	13	0	16	5	14
47	32	43	271	41	4	34	22	93	124	49	197	14	7	13	0	16	5	15
48	33	45	276	41	5	33	22	91	127	49	209	16	7	13	0	16	4	15
49	35	47	282	42	7	32	22	89	131	49	220	17	7	14	0	17	3	16
50	36	50	287	42	9	31	24	87	134	49	232	18	8	14	0	17	2	16

51	40	52	293	43	11	31	25	84	138	49	244	19	8	15	0	17	1	17
52	45	54	298	43	13	30	27	82	141	49	255	19	9	15	0	17	0	17
53	49	56	299	44	15	29	28	80	145	49	267	19	9	15	0	17	0	18
54	53	58	300	44	17	28	30	78	148	49	279	19	10	16	0	18	0	18
55	58	60	301	45	19	27	32	75	152	49	290	20	10	16	0	18	0	18
56	62	63	302	45	20	27	33	73	155	49	302	20	11	17	0	18	0	19
57	66	65	303	46	22	26	35	71	159	49	313	20	11	17	0	18	0	19
58	71	67	304	46	24	25	37	68	162	49	325	20	12	17	0	19	0	20
59	75	69	305	47	26	24	38	66	166	49	337	20	12	18	0	19	0	20
60	79	71	306	47	28	23	40	64	169	49	348	20	13	18	0	19	0	21
61	83	73	306	48	30	22	41	62	173	49	360	21	13	18	0	19	0	21
62	88	75	307	48	32	22	43	59	176	49	372	21	14	19	0	19	0	22
63	92	78	308	49	34	21	45	57	180	49	383	21	14	19	0	20	0	22
64	96	80	309	49	35	20	46	55	183	49	395	21	14	20	0	20	0	23
65	101	82	310	50	37	19	48	53	187	49	407	21	15	20	0	20	0	23
66	105	84	311	50	39	18	49	50	190	49	418	21	15	20	0	20	0	24
67	109	86	312	51	41	17	51	48	194	49	430	21	16	21	0	20	0	24
68	114	88	313	51	43	17	53	46	197	49	441	22	16	21	0	21	0	24
69	118	90	314	52	45	16	54	44	201	49	453	22	17	22	0	21	0	25
70	122	93	315	52	47	15	56	41	204	49	465	22	17	22	0	21	0	25
71	127	95	316	53	49	14	57	39	208	49	476	22	18	22	0	21	0	26
72	131	97	317	53	50	13	59	37	211	49	488	22	18	23	0	22	0	26
73	135	99	318	54	52	13	61	34	215	49	500	22	19	23	0	22	0	27
74	140	101	319	54	54	12	62	32	218	49	511	23	19	24	0	22	0	27
75	144	103	320	55	56	11	64	30	222	49	523	23	20	24	0	22	0	28
76	148	106	321	55	58	10	66	28	225	49	535	23	20	24	0	22	0	28
77	153	108	321	56	60	9	67	25	229	49	546	23	21	25	0	23	0	29
78	157	110	322	56	62	8	69	23	232	49	558	23	21	25	0	23	0	29
79	161	112	323	57	64	8	70	21	236	49	569	23	21	25	0	23	0	30
80	165	114	324	57	65	7	72	19	239	49	581	23	22	26	0	23	0	30
81	170	116	325	58	67	6	74	16	243	49	593	24	22	26	0	23	0	30
82	174	118	326	58	69	5	75	14	246	49	604	24	23	27	0	24	0	31
83	178	121	327	59	71	4	77	12	250	49	616	24	23	27	0	24	0	31